**Finite Element Analysis**

General Documentation

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# Introduction

Finite Element Analysis is a personal project centered around the development of a Python Finite Element Analysis software for personal learning.

# Physics

The following will provide a description on how all the math and physics used work. The fundamental theory behind the program is the **method of virtual work.** The reason that this method is so powerful is that many times in solving for complicated geometries or displacements there can be large sets of differential equations present, often times these are not solvable. Take for example Euler-Bernoulli beam theory, a simplified version of this may give the differential equation seen in Equation (1).

This can be remedied using the principle of virtual work. This is useful for finding approximate displacement functions. It comes from the basic idea that:

This is essentially impossible to visualize in real life as the virtual strains and virtual displacements are infinitesimal and a purely mathematical solution. For a continuum the Equation (2) provides the equilibrium equations. Note that the following equations are only the solutions for two dimensions. It is planned to expand to the third dimension for the solver later.

This is the standard way to write this, but I prefer to look at it in the following way as it makes it clearer what is happening without all the tensor notation, this way is seen in Equation (3).

Where b is the body force. Now we need to multiply the forces in the directions by the corresponding virtual displacement. For every quantity that is virtual a \* is placed beside it.

This can be generalized into the formula seen in Equation (5).

The following will now provide the overall derivation for the virtual work. In Equation (6) the partial derivative of an unrelated quantity is taken and rearranged to be substituted back into Equation (5). The first part comes from product rule.

This can then be substituted back into Equation (5), giving Equation (7).

This part is the cool part of the proof. We know that represents the Cauchy stress tensor we can show that the equation in Equation (8) holds true.

And because we know that through symmetry we can factor it out in Equation (9).

FINALLY, this gives Equation (10) which has the real stresses multiplied by the virtual strains, this is internal virtual work.

Now we can integrate over the domain giving us Equation (11).

We now need to fix the first term and then the derivation for virtual work is completed. We can start by stating that the first term is the divergence of the stress tensor and the displacement.

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